Exercise 1

Let's consider an optically pumped amplifier that amplifies light between levels E_1 and E_2. The population at ground level, level 1, and level 2 are denoted N_g, N_1, N_2, respectively. No signal is at the input.

Assume that $R_1 = 0$ and that R_1 is realized by exciting atoms from the ground level to level 2 using photons of frequency E_2/h. These photons are absorbed with a transition probability W_p (i.e. probability between ground level and level 2)

Assume also that $\tau_2 \approx \tau_{sp}$ and $\tau_1 \ll \tau_{sp}$. In steady state we have $N_1 \approx 0$ and $N_0 \approx R_2\tau_{sp}$.

If N_a is the total population distributed over levels 0,1 and 2:

(a) Write the steady state equation for population N_2.

The photon rate equation includes the pumping rate R_2 and spontaneous emission rate N_2/τ_2. Since $\tau_2 \approx \tau_{sp}$, we have:

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_{sp}}$$

In steady state (on more time evolution), this becomes:

$$R_2 = \frac{N_2}{\tau_{sp},}$$

(b) Write the rate equation for the pumping rate R_2 as a function of W_p and show that $R_2 \approx (N_a - 2N_0)W_p$. (hint: R_2 will depend on the population N_g and N_2)

Considering that R_2 is realized by exciting atoms from the ground level to level 2, and that photons are absorbed with a transition probability W_p between the ground and level 2. There is also emission from level 2 with the same transition probability:

$$R_2 = W_pN_g - W_pN_2$$

In the steady state condition, there is no population in level 1 ($N_1 \approx 0$), hence the population $N_0 = N_2 - N_1 \approx N_2$. The total population is now $N_a = N_g + N_2$, and we finally get that $N_g \approx N_a - N_0$.

We have that $R_2 = W_p(N_a - N_0) - W_pN_0 = W_p(N_a - 2N_0)$

(c) Show that $N_0 \approx N_a\tau_{sp}\frac{W_p}{(1+2\tau_{sp}W_p)}$

To obtain an expression for N_0, we replace the expression for R_2 obtained at part (b) in the equation derived in part (a). Therefore, in the steady state condition we have:

$$W_p(N_a - 2N_0) = \frac{N_2}{\tau_{sp}}$$

$$W_p(N_a - 2N_0) = \frac{N_0}{\tau_{sp}}$$

$$2W_pN_0 + \frac{N_0}{\tau_{sp}} = W_pN_a$$

$$N_0 = N_a\tau_{sp}\frac{W_p}{1 + 2W_p\tau_{sp}}$$

Note that it is always desirable to obtain population difference in terms of total population. Such an expression shows the ratio of the inverted photons to total photon number which is equivalent to the efficiency of the pumping scheme.

Exercise 2

We are building and EDFA with a fiber length of 30 m. The stimulated emission cross section for this specific doped fiber is $\sigma(\nu) = 5 \cdot 10^{-25}\ \mathrm{m}^2$.

What population difference $(N_2 - N_1)$ is necessary for the amplifier gain to reach 30 dB?

Neglecting gain saturation, the amplifier gain G in a doped fiber of length L is given by :

$$G = \exp[\gamma(\nu)L] = \exp[(N_2 - N_1)\sigma(\nu)L]$$
$$1000 = \exp[(N_2 - N_1)(5 \cdot 10^{-25})30]$$

We get that $(N_2 - N_1) = 4.6 \cdot 10^{23}\mathrm{m}^{-3}$

Exercise 3

In a DWDM system using PSK modulation one wishes to transport data over L_T = 1000 km with as few as possible wavelength channels. In the system one also wishes to minimize the number of in-line EDFAs (N_A) used in the link. The fiber loss is α =0.2 dB/km.

The required OSNR for a given total bit rate scales approximately linearly with the number of bits/symbols, e.g. QPSK (2 bits/symbol) needs twice the OSNR required for binary PSK (1 bit/symbol).

(a) The binary PSK system needs ten amplifiers to satisfy the OSNR requirement at the end of the link. What is in this case the distance L_A between amplifiers ? What is the gain G_1 required of the EDFA in this case?

Given the loss is α =0.2 dB/km, the total fiber length of 1000 km and the need for 10 amplifiers: the distance between amplifiers should be 100 km and therefore the gain $G_1 = 100(0.2) = 20$ dB.

(b) Show that the number of amplifiers N_A can be expressed as $N_A = (\alpha L_T)/(\ln G)$ and that the ONSR at the end of the link can be written as :

$$OSNR_{end} = \frac{P_{in}\ln G}{2n_{sp}h\nu\Delta\nu_0\alpha L_T(G-1)}$$

We know that $G = \exp(\alpha L_A)$ so $\alpha L_A = \ln G$. Since $L_T = N_A L_A$, then :

$$N_A = \frac{(\alpha L_T)}{(\ln G)}$$

We use the expression seen in class and replace N_A by the previous expression to get that :

$$OSNR_{end} = \frac{P_{in}\ln G}{2n_{sp}h\nu\Delta\nu_0\alpha L_T(G-1)}$$

(c) Using this expression of OSNR and assuming that the optical filter bandwidth is the same in all cases, what should be the gain G_1 and G_4 required of the EDFA if the linked used QPSK (2 bits/symbol) or 16-QAM (4 bits/symbol), respectively? (HINT: start by expressing the ratio of the required OSNR compared to the PSK case, which simplifies many of the terms in common …)

Using the previous expression, cancelling all common terms ($P_{in}, 2n_{sp}h\nu\Delta\nu_0, L_T$), we get:

$$\frac{OSNR\ (2bits/symbol)}{OSNR\ (1\ bit/symbol)} = \frac{lnG_2}{(G_2-1)}\frac{(G_1-1)}{lnG_1} = 2$$

$$\frac{OSNR\ (4bits/symbol)}{OSNR\ (1\ bit/symbol)} = \frac{lnG_4}{(G_4-1)}\frac{(G_1-1)}{lnG_1} = 4$$

Given that $G_1 = 20$ dB, we deduce that $\frac{(G_1-1)}{lnG_1} = 21.5$ and therefore we get $G_2 = 16.1$ dB and similarly $G_4 = 12$ dB.

(d) How many amplifiers would be needed for the link if QPSK or 16-QAM was used?

For such values of gain and the length/loss, the corresponding amplifier spacing will need to be no more than 80.5 km for QPSK and 60 km for 16-QAM. We would therefore need 13 amplifiers for QPSK and 17 for 16-QAM.

Exercise 4

We want to set up a 200 km link at 1550 nm operating at 1 Gb/s using RZ format. The transmitter has a given rise time of 100 ps. The receiver is RC limited and has a capacitance of 2 pF and a load resistance of 80 Ω. Dispersion is 17 ps/km.nm. The source has a spectral width of 0.016 nm.

From the rise time budget, can such link operate correctly? What are the possible solutions ?

The rise time budget has three components:

- Transmitter rise time is given : $T_{tr} = 100 \text{ ps}$
- Receiver rise time: $T_{rec} = R_L C = 160\ ps$
- Fiber rise time: $T_{fiber} \approx L|D|\Delta\lambda = 54.4\ ps$

The overall system's rise time is : $T_t = \sqrt{T_{tr}^2 + T_{rec}^2 + T_{fiber}^2} = 196.4\ ps$

The system's bandwidth is therefore approximately 1.84 GHz. Since this bandwidth is more than the RZ required bandwidth of 1 GHz, the system can work. (we can also see that the rise time is less than 35 % of the RZ bit period of 1 ns).

Exercise 5

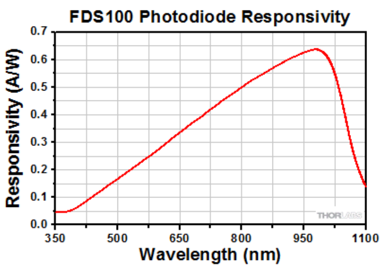
We are designing a typical digital fiber optic link. The data to be transmitted is operating at a data rate of 18 Mb/s with NRZ and we must guarantee a BER of 10^{-9}.

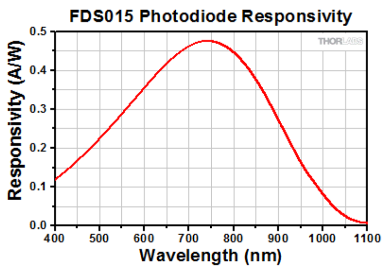
We are using an AlGaAs LED emitting at 850 nm and 100 mW of power can be coupled into a multimode GRIN fiber with a core of 50 μm. The source including its drive circuit has a rise time of 12 ns. The GRIN has a loss of 2.5 dB/km and given the spectral bandwidth of the source induces a combined material/intermodal-group-delay dispersion of 4 ns/km.

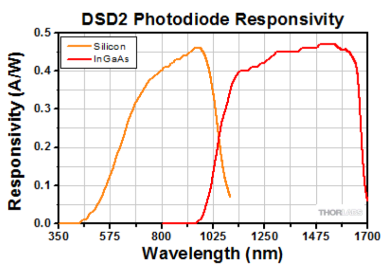
The fiber requires splicing every 1 km with a 0.5 dB/splice of loss. We also need 2 connectors, one at the transmitter and one at receiver end, each with 1 dB of loss.

The receiver uses a p-i-n photodiode with a load resistance of 1 kΩ, a noise figure $f_n = 3$ dB and with a bandwidth assumed to be exactly matched for 20 Mb/s NRZ data. Assume a temperature of 300 K and that the system is in the thermal limit

(a) You have a choice between the p-i-ns shown below. Assume they all have a response time of 10 ns Which one do you use and why?







We take FDS100 as it has the highest responsivity of R = 0.56 A/W at our operating wavelength.

(b) Based on the photodiode you picked, what is the receiver sensitivity for BER =10^{-9} ?

$$P_{rec} = \frac{Q\sigma_T}{R}$$

The thermal noise variance is (with assuming $\Delta f = \frac{B}{2} = 10\ MHz$)

$$\sigma_T^2 = \frac{4k_B T F_n}{R_L} \Delta f$$
$$\sigma_T^2 = \frac{4(1.38\ 10^{-23})(300)(2)}{1000} 10\ 10^6$$
$$\sigma_T = 1.81\ 10^{-8}\ A$$

For BER of 10^{-9} (Q = 6), the receiver sensitivity is therefore:

$$P_{rec} = \frac{Q\sigma_T}{R} = 0.195\ \mu W$$

(c) Verify that your assumption of thermal limit is correct.

For such received power let's calculate shot noise variance:

$$\sigma_s^2 = 2qR(2\overline{P}_{rec})\Delta f$$
$$\sigma_s^2 = 2(1.602\ 10^{-19})0.56(2)(0.195\ 10^{-6})10\ 10^6$$
$$\sigma_s = 8.36\ 10^{-10} A$$

We are indeed thermal noise limited.

(d) What is the possible maximum loss limited link length without repeaters if a 1 dB margin is required?

We do a power budget:

$$\bar{P}_{tr} - P_C - P_m - \alpha L_{max} = \bar{P}_{rec}$$
$$10\log_{10}(0.1) - 2 - 1 - 0.5(L_{max} - 1) - 2.5L_{max} = 10\log_{10}(1.95\ 10^{-4}\ )$$
$$-12.5 - 3L_{max} = -37.1$$
$$-12.5 - 3L_{max} = -37.1$$
$$L_{max} = 8\ km$$

(e) Based on rise time budget, is this system viable?

We do a rise time budget:

$$T_r^2 = T_{tr}^2 + T_{fiber}^2 + T_{rec}^2$$
$$T_r = \sqrt{(12\ 10^{-9})^2 + (10\ 10^{-9})^2 + (8 \times 4\ 10^{-9})^2}$$
$$T_r = 35.6\ ns$$

The bandwidth is therefore $\Delta f = \frac{0.35}{T_r} = 9.82\ MHz$

For NRZ at 18 Mb/s we need a bandwidth of 9 MHz so the system is viable

(f) If the system is viable, how much further could you actually go? in this case what would be the BER at the end of your link ?

The limit in terms of bandwidth is: $T_r = \frac{0.35}{9\ 10^6} = 38.89\ ns$
We could increase the distance to L_{BW_limit}:

$$T_r = \sqrt{(12\ 10^{-9})^2 + (10\ 10^{-9})^2 + \left(L_{BW\_limit} \times 4\ 10^{-9}\right)^2}$$

$$L_{BW\_limit} = 8.9\ km$$

In this case the power at the receiver would be (from the power budget):

$$-12.5 - 3L_{BW\_limit} = \bar{P}_{rec}$$

$$-12.5 - 3L_{BW_{limit}} = \bar{P}_{rec} = -39.2\ \text{dBm} = 0.12\ \mu\text{W}$$

In this case the Q would be:

$$Q = \frac{RP_{rec}}{\sigma_T} = 3.72$$

And BER would be about 10^{-4}.

Exercise 6

You are designing a 1550 nm 3000 km long fully dispersion-compensated fiber-optic link. The link is periodically amplified, and you are considering either 50 km or 75 km spacing between amplifiers. The dilemma is that while an increased span length saves you a lot of money on EDFAs, the OSNR will be worse. The fiber average loss is 0.3 dB/km because of dispersion compensation. Each span is followed by an EDFA with gain equal to span loss. The EDFA noise figure is 5 dB and the average input signal power into each span is 1 mW.

(a) Calculate the OSNR (at the receiver) in the two cases. Assume 0.1 nm of optical filter bandwidth is sufficient.

We know that: $OSNR = P_{in} - N_A - 2n_{sp} + 58$

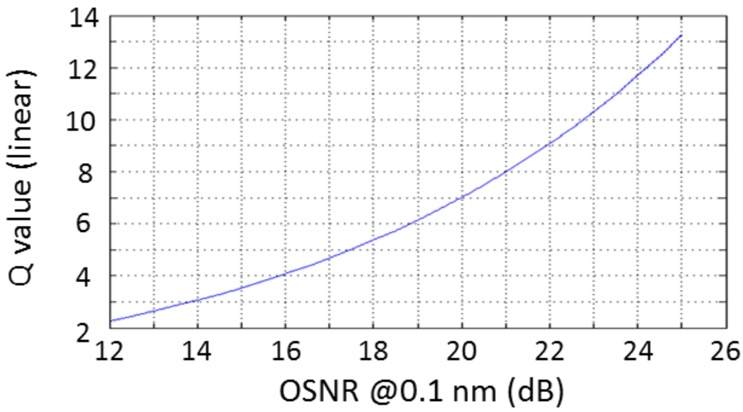
For $L_A = 50$ km, the number of required amplifiers for the 3000 km full link is $N_A = 60 = 17.78$ dB. Hence:

$$OSNR_{50k} = 0 - 17.78 - 5 - (0.3)(50) + 58 = 20.22$$

For $L_A = 75$ km, the number of required amplifier for the 3000 km full link is $N_A = 40 = 16.02$ dB. Hence:

$$OSNR_{75k} = 0 - 16.02 - 5 - (0.3)(75) + 58 = 14.48\ \text{dB}$$

(b) Use the figure below, which shows the relation between OSNR and Q-value and determine the BER with the two span lengths.



For L_A =50 km, from the graph Q ~7; the BER is therefore BER ~ 1.3 10^{-12}

For L_A =75 km, from the graph Q ~3.5; the BER is therefore BER ~ 2.5 10^{-4}

(c) Given that you need -22.5 dBm of power at the receiver, what could be the maximum length of an additional last fiber span, after the last EDFA (Reuse the fact that the input power in the last span is 1 mW).

We do an additional power budget: $-22.5 = 0 - (0.3)L_{end}$

Therefore $L_{end} = \frac{22.5}{0.3} = 75\ \text{km}$

(d) You want to increase the reach by adding backward Raman pumping on the last span. You have a 1 W Raman pump available. The loss at the pump wavelength is 0.25 dB/km. The fiber has $a_p = 50\ \mu\text{m}^2$ and $g_r = 6 \cdot 10^{-14}\ \text{m/W}$. How long could this last span be? Assume the length of the last span is much longer than the effective length.

We assume $L_{end} \gg L_{eff}$ therefore $L_{eff} \approx \frac{1}{\alpha_p} = \frac{4.343}{0.25} = 17.372\ \text{km}$

We want that:

$$
\begin{aligned}
10^{-2.25} &= \exp\left[g_R P_0 \frac{L_{eff}}{a_P} - \alpha_S L \right] \\
5.62 \cdot 10^{-3} &= \exp\left[6 \cdot 10^{-14} \frac{17382}{50 \cdot 10^{-12}} - \alpha_S L \right] \\
-5.18 &= 20.85 - \alpha_S L \\
L &= 376.85\ \text{km}
\end{aligned}
$$

By adding Raman amplification, the last span could be of 376.85 km instead of 75 km.